Determinacy in a Canonical New Keynesian Model under a Money Growth Rule

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Introduction

What should be the monetary policy instrument?

- Nominal interest rate: Taylor rule
- Money growth: Friedman's k rule

Important aspect: **Determinacy**

Otherwise:

- Coordination on a *bad equilibrium*
- Self fulfilling fluctuations

Richard Clarida, Jordi Galí and Mark Gertler (2000)

In the pre-Volcker years the Fed typically raised nominal interest rates by less than any increase in expecting inflation, thus letting real short term rates decline as anticipated inflation rose.

...

Finally, we have argued that the pre-Volcker rule may have contained the seeds of macroecnomic instability that seemed to characterize the late sixties and seventies. In particular, in the context of a calibrated sticky price model, the pre-Volcker rule leaves open the possibility of bursts of inflation and output that result from self-fulfilling changes in expectations.

This paper

I study the determinacy of a *Canonical New Keynesian model under a Money Growth Rule* under different timings in the introduction of money.

Previous Studies:

- Interest rate rules: Bullard and Mitra (2002)
- Money Growth rules
  - Classic: Calstrom and Ferst's (2003) analytical analysis
  - New Keynesian: Galí's (2015), Chapter 3, numerical analysis for *Cash-when-I'm-done* timing

HERE: Analytical results for the New Keynesian model

Model set-up

$$\max_{\{C_t, A_t, N_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{(A_t/P_t)^{1-\nu} - 1}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right\}$$

$$s.t. \quad P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t + T_t$$

- Cash-when-I'm-done (CWID) money demand: $A_t = M_t$

$$m_t - p_t = \frac{\sigma}{\nu} y_t - \eta i_t$$

- Cash-in-advance (CIA) money demand: $A_t = M_{t-1}$

$$m_t - p_t = \frac{\sigma}{\nu} y_t - \eta i_t - \left(\frac{1-\nu}{\nu} \right) E_t\{\pi_{t+1}\}$$

Encompasses both transaction cost and shopping time models, Feenstra (1986). We set $\nu = \sigma$, as it implies the natural assumption of unit elasticity with respect to income.

- Aggregate Demand: the Dynamic IS equation

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\tilde{y}_{t+1}\}$$

- Aggregate Supply: the New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t = \sum_{k=0}^{\infty} \beta^k E_t\{\tilde{y}_{t+k}\} = -\lambda \sum_{k=0}^{\infty} \beta^k E_t\{\hat{u}_{t+k}\}$$

- Money growth rule

$$\Delta m = 0$$
$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t$$

where $l_t \equiv m_t - p_t$ and "^" on top of a variable denotes it's deviation from the steady state.

System

$$\mathbf{A_{M,0}} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \hat{r}_t^n \\ \hat{y}_t^n \\ \Delta m_t \end{bmatrix}$$

where:

$$\mathbf{A_{M,0}} = \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}; \mathbf{A_{M,1}} = \begin{bmatrix} \sigma\eta & \Omega & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}; \mathbf{B_M} = \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

and $\Omega \equiv \eta$ under CWID timing and $\Omega \equiv \frac{1-\sigma}{\sigma} + \eta$ for CIA timing.

Determinacy I

For the system to be determinate, we need $\mathbf{A_M} \equiv \mathbf{A_{M,0}}^{-1}\mathbf{A_{M,1}}$ to have one eigenvalue outside (or on) the unit circle and two inside, based on Blanchard and Khan (1980).

$$\mathbf{A_M} = \begin{bmatrix} \frac{\sigma\eta}{1+\sigma\eta} & \frac{\eta}{1+\sigma\eta} & \frac{1}{1+\sigma\eta} \\ \frac{\kappa\sigma\eta}{1+\sigma\eta} & \frac{\kappa\eta}{1+\sigma\eta}+\beta & \frac{\kappa}{1+\sigma\eta} \\ \frac{\kappa\sigma\eta}{1+\sigma\eta} & \frac{\kappa\eta}{1+\sigma\eta}+\beta & \frac{\kappa}{1+\sigma\eta}+1 \end{bmatrix}$$

whose eigenvalues are given by the roots of the following polynomial:

$$p(x) = \underbrace{-}_{a} x^3 + \underbrace{\left(\frac{\sigma\eta+\kappa(1+\Omega)}{1+\sigma\eta}+1+\beta\right)}_{b} x^2 \underbrace{-\left((1+\beta)\frac{\sigma\eta}{1+\sigma\eta}+\frac{\kappa\Omega}{1+\sigma\eta}+\beta\right)}_{c} x + \underbrace{\frac{\beta\sigma\eta}{1+\sigma\eta}}_{d}$$

where $\sigma > 0$; $\eta > 0$; $\kappa > 0$ and $\beta \in (0,1)$.

Determinacy II: $\mathbb{R}$

1. $p(+\infty) < 0$
2. $p(1) = \kappa/(1 + \sigma\eta) > 0$

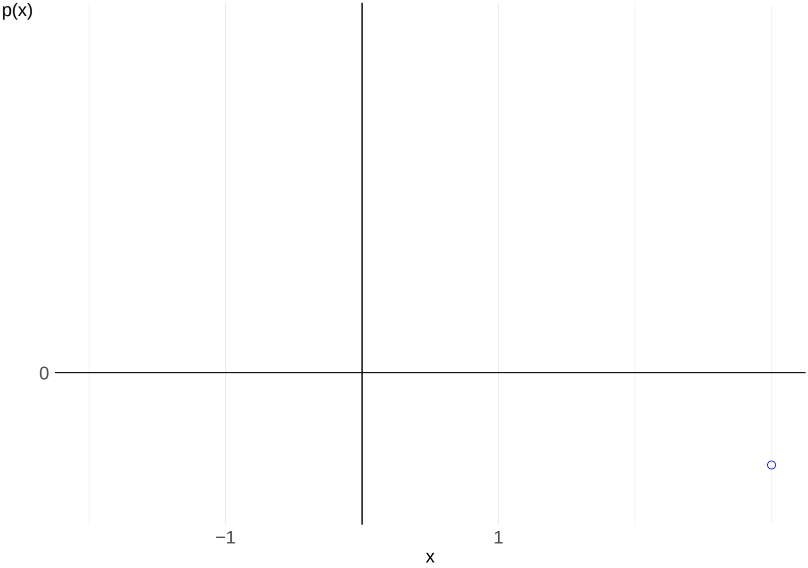
- From 1 and 2, by continuity and Bolzano's theorem, there must exist at least (only) one eingenvalue, λ_1, larger than 1.

3. $p(x) > 0 \;\; \forall \; x \in (-\infty, -1] \Leftrightarrow p(-1) > 0$

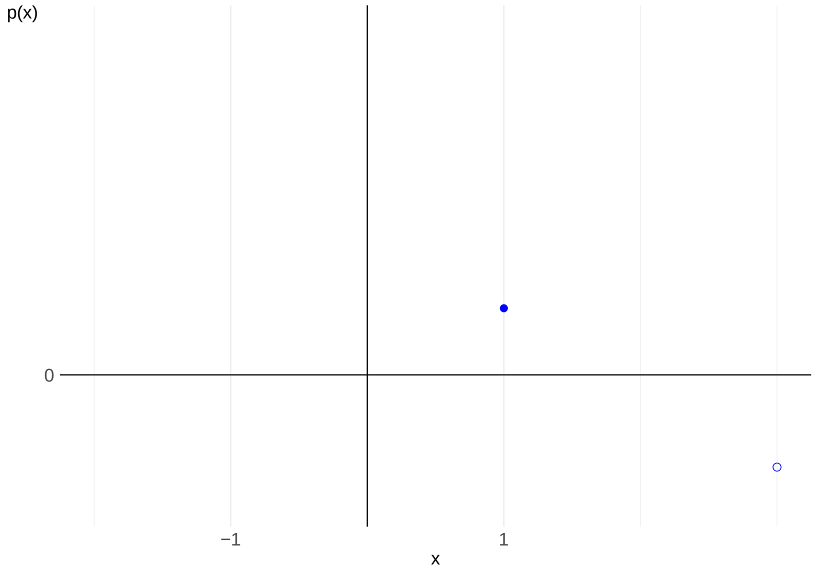
- Therefore, the other two eigenvalues, λ_2 and λ_3, must be inside the unit circle when:

$$\Omega > -\frac{1}{2} - (1+\beta)\frac{\sigma\eta}{\kappa} - \frac{(1+\sigma\eta)(1+\beta)}{\kappa} \quad \star$$

  Since $p(-\infty) > 0$, if $\star$ is not met, $p(-1) < 0$, there are two eigenvalues outside the unit circle, and the solution consists of three real roots.



p(x)
0
−1
1
x


p(x)
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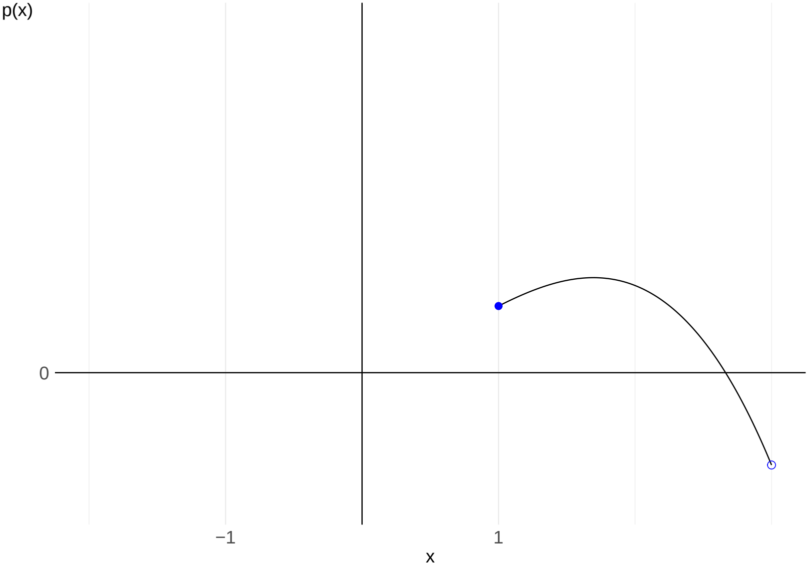

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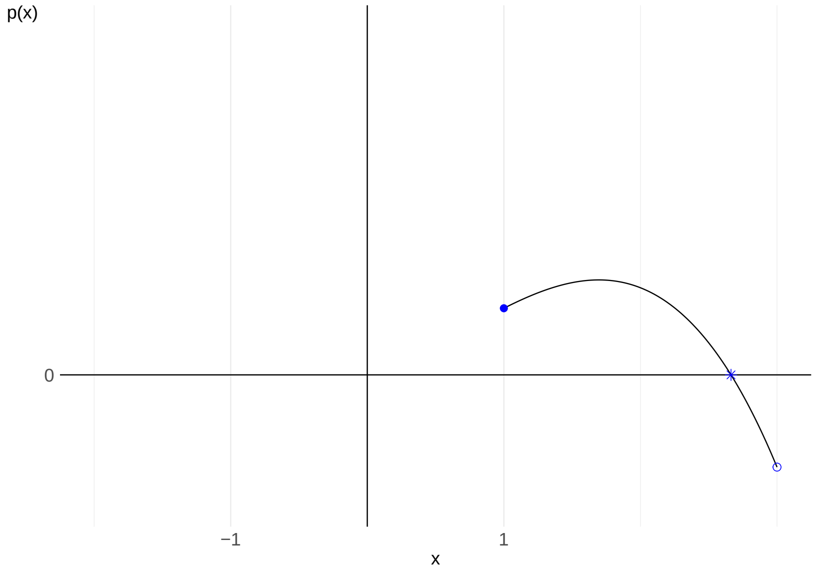
x



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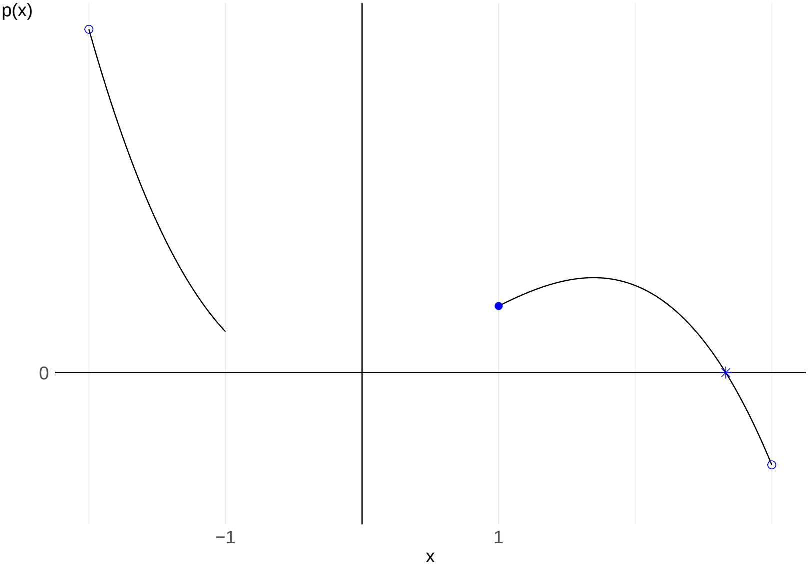

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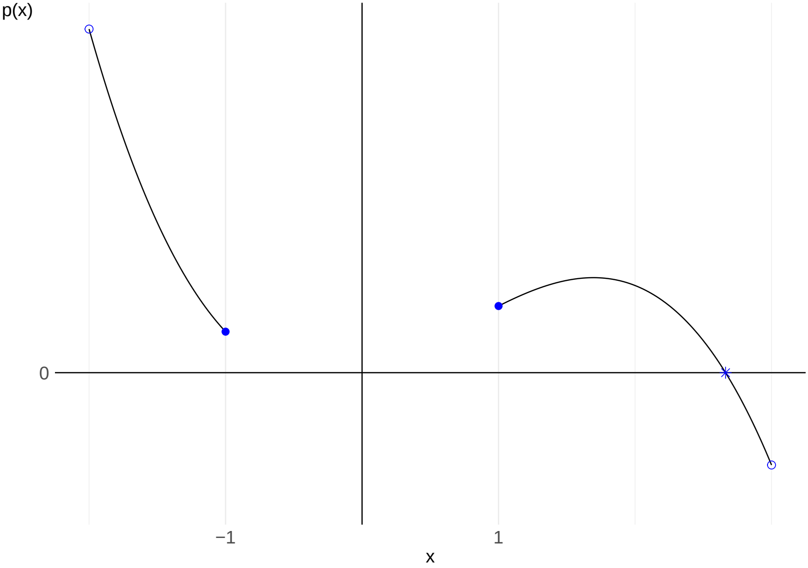
x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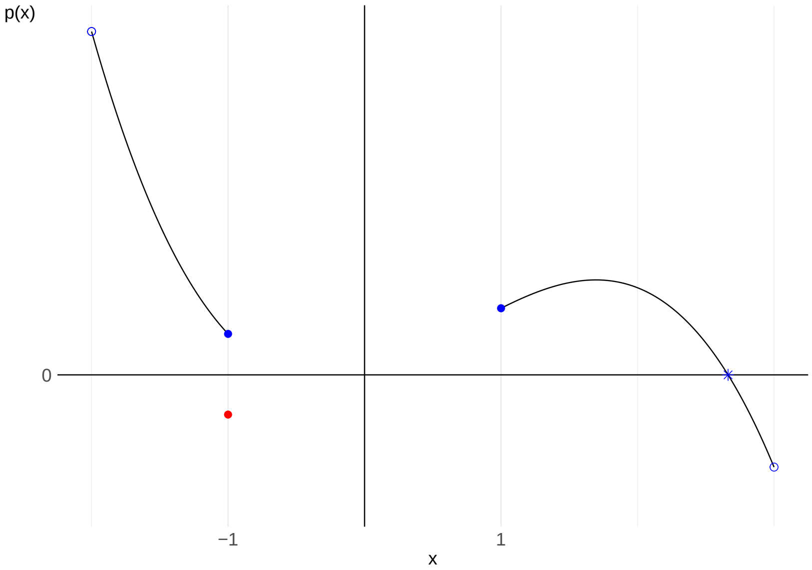
x


p(x)
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−1
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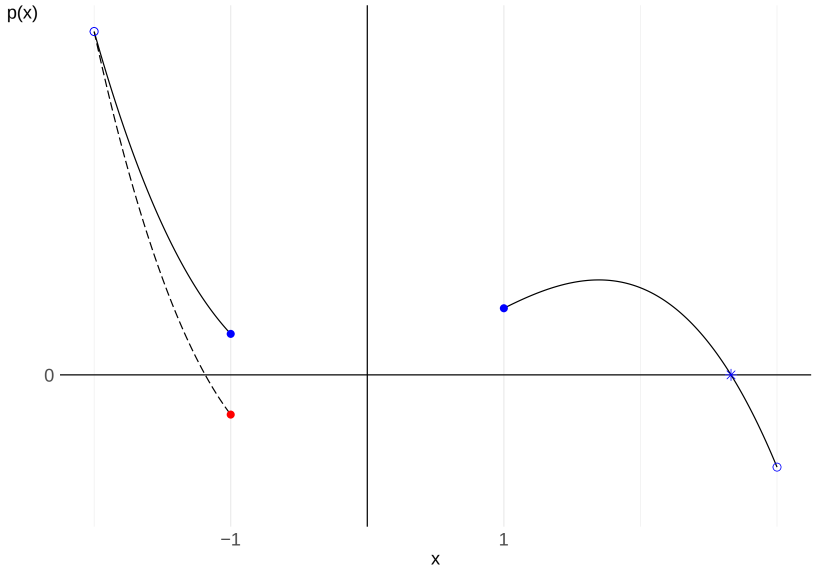

p(x)
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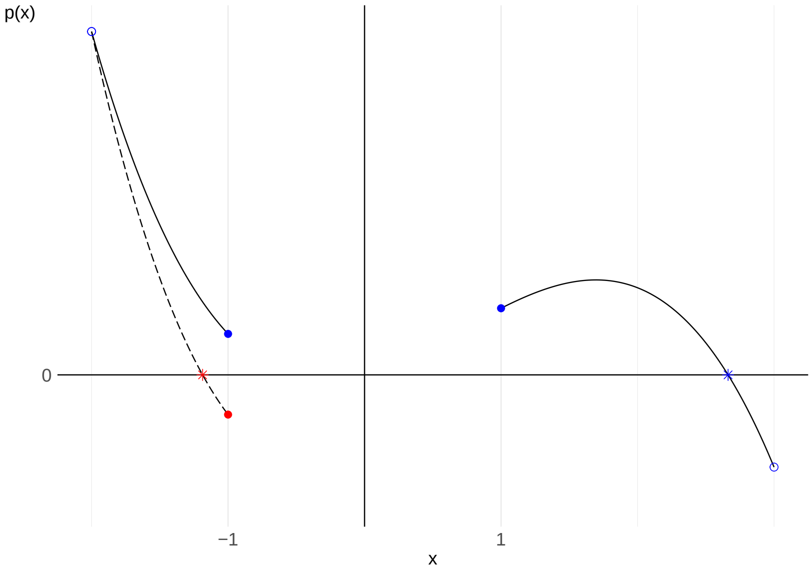
x



p(x)
0
−1
1
x

Determinacy III: Complex Conjugates

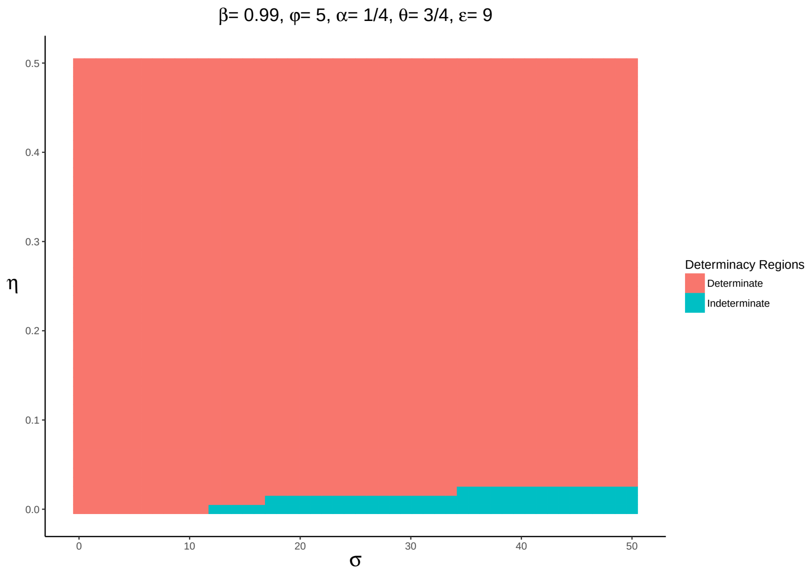
To adress the case of two (λ_2 and λ_3) complex conjugates, we write our polynomial in a more general form:

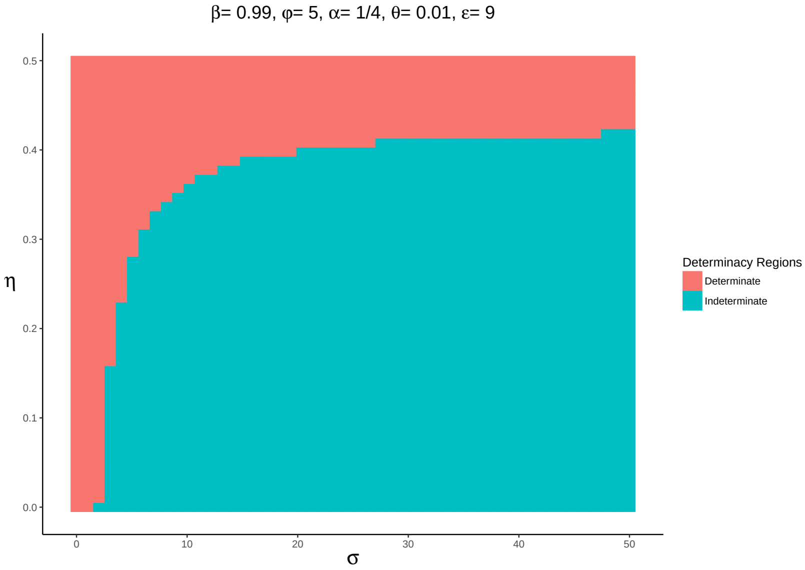
$$p(x) = -(x-\lambda_1)(x-\lambda_2)(x-\lambda_3) = -x^3 + bx^2 + cx + d$$

where $d = \lambda_1\lambda_2\lambda_3$ is the independent component. Since $|\lambda_2| = |\lambda_3|$ and $\lambda_2\lambda_3 = d\frac{1}{\lambda_1} \rightarrow |\lambda_3|^2 = d\frac{1}{\lambda_1}$.

$$d\frac{1}{\lambda_1} = \underbrace{\beta}_{\in(0,1)} \underbrace{\frac{\sigma\eta}{\lambda_1 + \lambda_1\sigma\eta}}_{<1 \ due \ to \ \lambda_1>1} < 1$$

Therefore, one root is on or outside the unit circle (λ_1) and two (λ_2 and λ_3) inside the unit circle. The system is always determinate for the case of two complex conjugates.


β= 0.99, φ= 5, α= 1/4, θ= 3/4, ε= 9
η
0.5
0.4
0.3
0.2
0.1
0.0
0
10
20
30
40
50
σ
Determinacy Regions
Determinate
Indeterminate



β= 0.99, φ= 5, α= 1/4, θ= 0.01, ε= 9
0.5
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0.0
η
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Determinacy Regions
Determinate
Indeterminate

Discussion

To obtain indeterminacy we need:

1. *Cash-in-advance* timing
2. A high degree of risk aversion ($\sigma \uparrow$)
3. A low interest semi-elasticity of money demand ($\eta \downarrow$)
4. A sufficiently high degree of price flexibility ($\kappa \uparrow$)

- As we get close to the case of fully flexible prices, $\kappa \to \infty$, *4* is no longer needed, leading to the conditions obtained by Carlstrom and Fuerst (2003) in a flexible price economy.
- In an economy with sticky prices, an exogenous money growth rule may deliver a determinate equilibrium even when it would not under flexible prices. As $\kappa \downarrow$, the parameter range yielding determinacy under CIA timing expands. Still, the rule seems robust under reasonable calibration.
- In the limit case of fix prices, $\kappa = 0$, the system is always determinate.

Thank You